Quantum Teleportation without Irreversible Detection:
NMR-Experiment

Yasushi Kondo

Department of Physics, Kinki University, Higashiosaka, Osaka 577-8502
(Received June 2, 2007; accepted July 25, 2007; published September 25, 2007)

We implement a quantum teleportation algorithm in NMR with $^{13}$C-labeled L-alanine. Here three $^{13}$C-nuclear spins are qubits and H-nuclear spins are employed as a well-defined quantum mechanical interface to a detector. Our experiment is considered as a realization of “quantum teleportation without irreversible detection” and is helpful for understanding the principle of quantum teleportation.

KEYWORDS: quantum teleportation, NMR, quantum computer, alanine
DOI: 10.1143/JPSJ.76.104004

1. Introduction

The quantum teleportation\(^1\) of an unknown quantum state is a surprising demonstration,\(^2,3\) since it may be intuitively thought to violate the uncertainty principle of quantum mechanics, which forbids extracting all the information in an quantum object. By employing a reversible system and the environment and how it recovers from information is hidden within the correlations between the information in a quantum object. By employing a reversible quantum mechanics, which forbids extracting all the intuitively thought to violate the uncertainty principle of 4) More general discussions were made by Nielsen and Caves.\(^5\)

We have been employing a liquid-state NMR quantum computer, hereafter called an NMR-QC, as a test bench for a more realistic quantum computer. For example, we have demonstrated our time-optimal implementation of two-qubit quantum algorithms\(^6\) and performed experiments where artificial relaxations are generated and suppressed by means of bang-bang control.\(^7\) The purpose of this paper is (i) to realize Braunstein’s reversible detector in a quantum teleportation algorithm and confirm his discussions with a NMR-QC experiments.

Section 2 is a brief review of the theory of quantum teleportation,\(^1,4\) the entanglement fidelity\(^8\) (a measure of how well quantum information is preserved). In §3 we describe details of our experiments. There we discuss the sample and our spectrometer, the Hamiltonian of L-alanine (our quantum computer), how to realize a detector with the Hamiltonian, the pulse sequence realizing quantum teleportation and the experimental results. Section 4 is devoted to summary and conclusions.

2. Theory

2.1 Quantum teleportation without irreversible detection

We briefly review the discussion by Braunstein\(^9\) in which he employed a reversible detector. In order to make our discussions as simple as possible, we limit ourselves to the case of two-state system teleportation in this paper. See the original paper\(^10\) for a general N-state system case.

The original quantum teleportation scheme\(^1\) is as follows. Qubit 1 in an unknown state $|\psi\rangle$ is jointly measured with qubit 2, which forms an entangled pair $|\beta_{00}\rangle$ with qubit 3, and then qubit 3 is unitary transformed according to the result of the joint measurement and becomes $|\psi\rangle$. The mathematics behind the quantum teleportation is summarized by the decomposition of the initial state $|\psi\rangle \otimes |\beta_{00}\rangle$,

$$|\psi\rangle \otimes |\beta_{00}\rangle = \frac{1}{2} \sum_{i=0}^{\pi} |\beta_{ij}\rangle \otimes U_{ji}^\dagger |\psi\rangle,$$

where

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), \quad |\beta_{01}\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle),$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle), \quad |\beta_{11}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle),$$

and

$$U_{00}^\dagger = I, \quad U_{01}^\dagger = \sigma_x, \quad U_{10}^\dagger = \sigma_z, \quad U_{11}^\dagger = \sigma_z \sigma_x.$$

Here, $\sigma_k (k=x,y,z)$ are standard Pauli matrices and $I$ is the identity matrix of dimension 2. After a joint measurement, the system becomes one of the four states, for example, $|\beta_{ij}\rangle \otimes \sigma_k |\psi\rangle$ and the information “01” is obtained. Then, operating $\sigma_k$, which corresponds to the information “01”, on qubit 3 leads the state $|\psi\rangle$.

Braunstein introduced a detector, that is labeled $ij$ and a $2^2$-states system.\(^4\) Now, the total initial state is

$$(00) \otimes |\psi\rangle \otimes |\beta_{00}\rangle,$$

including the detector state $|00\rangle$. The joint measurement in the quantum teleportation operation is described with a unitary operator $U_{\text{meas}}$,

$$U_{\text{meas}} = \sum_{i,j=0}^{\pi} \Sigma_{ij} |\beta_{ij}\rangle \langle \beta_{ij}| \otimes I,$$

where $\Sigma_{ij}$ operates only the detector and

$$\Sigma_{ij} = \begin{cases} 1 & \text{if } i = 0, j = 0 \\ 01 \langle 00 | + |01\rangle \langle 01 | + |10\rangle \langle 10 | + |11\rangle \langle 11 | & \text{if } i = 0, j = 1 \\ |10\rangle \langle 00 | + |01\rangle \langle 01 | + |00\rangle \langle 10 | + |11\rangle \langle 11 | & \text{if } i = 1, j = 0 \\ |11\rangle \langle 00 | + |01\rangle \langle 01 | + |10\rangle \langle 10 | + |00\rangle \langle 11 | & \text{if } i = 1, j = 1 \end{cases}$$

E-mail: kondo@phys.kindai.ac.jp
After the system interacts with the detector, or after the “measurement”, the total state becomes
\[
\frac{1}{2} \sum_{i,j} |ij\rangle \otimes |\beta_i\rangle \otimes U_{ij}^\dagger |\psi\rangle.
\]
When the detector decoheres to a certain state $|ij\rangle$, it is the same as the conventional quantum teleportation scheme.

However, even if the detector does not decohere, applying the unitary operator $\sum_{i,j} |ij\rangle \otimes |\beta_i\rangle \otimes U_{ij}$ gives
\[
\left( \frac{1}{2} \sum_{i,j} |ij\rangle \otimes |\beta_i\rangle \right) \otimes |\psi\rangle.
\]

The above equation implies that an unknown quantum state teleportated from qubit 1 to qubit 3 without the irreversible amplification of an intermediate detector.

2.2 Quantum circuit

We will discuss a quantum circuit that realizes quantum teleportation. Brassard, Braunstein, and Cleve proposed the following quantum circuit, shown in Fig. 1.\(^{10}\) It is easy to confirm that the output of qubit 3 is the same as the input of qubit 1 for any unknown state.

The quantum circuit shown in Fig. 1 consists of three blocks. The left block $A$ makes an entangled pair $|\beta_{00}\rangle$ out of $|00\rangle$. The middle block $B$ converts $|\beta_{00}\rangle$, $|\beta_{10}\rangle$, $|\beta_{01}\rangle$, and $|\beta_{11}\rangle$ to $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, respectively. The right block $C$ is a circuit which realizes a conditional unitary gate for recovering the unknown state $|\psi\rangle$. Even when qubit 1 and 2 are measured after the middle block $B$, the output of the circuit does not change $^{10,11}$, and thus the unknown state $|\psi\rangle$ appears as the output of qubit 3. Therefore, the quantum circuit shown in Fig. 1 is considered as a circuit that realizes quantum teleportation with standard quantum gates in quantum computation.$^{10}$

Nielsen, Knill, and Laflamme performed the first complete quantum teleportation experiment by using an NMR-QC.$^{3}$

The quantum circuit shown in Fig. 1 was implemented and decoherence of qubit 1 and 2 between the blocks $B$ and $C$ was interpreted as an observation or measurement by the environment.$^{12}$

The quantum teleportation circuit including a detector, which was discussed by Braunstein,$^{4}$ is shown in Fig. 2. The detector qubits (qubit $d1$ and $d2$) and qubits 1 and 2 are entangled in the central block $M$ of the circuit. When qubit $d1$ and $d2$ decohere, qubits 1 and 2 decohere, too. We note that qubit 3 becomes the unknown state $|\psi\rangle$ regardless whether the detector qubits (qubit $d1$ and $d2$) decohere or not.

The essence of the “detector” discussed above is a capability of entanglement between qubits and an environment (a detector). Therefore any unitary operation, which can entangle the qubits and the environment, may be employed for introducing a “detector” into the quantum teleportation circuit. The following unitary operation is such an example, which is convenient for performing experiments.

\[
U_c(\theta_1, \theta_2) = \exp[-i(\theta_1 I_{zz}^d + \theta_2 I_{zz}^q)]
\]

where
\[
I_{zz}^{d, q} = I_x \otimes I \otimes I_x \otimes I_y \otimes I,
\]

Here $I_{zz} = \sigma_k / 2$ ($k = x, y, z$). Note that $U_c(4\pi - \theta_1, 4\pi - \theta_2)U_c(\theta_1, \theta_2)$ is the identity operator, or $U_c(\theta_1, \theta_2)$ is reversible.

2.3 Entanglement fidelity

The entanglement fidelity $^{8}$ is often employed for evaluating quantum processes, such as quantum teleportation, which in turn requires quantum process tomography.

A general state change in quantum mechanics is described by a map,

\[
\rho \rightarrow \frac{\mathcal{E}(\rho)}{\text{Tr}[\mathcal{E}(\rho)]},
\]

which connects an input state $\rho$ and an output state $\mathcal{E}(\rho)$. The general form for $\mathcal{E}$, which satisfies (1) linearity, (2) trace decreasing, (3) preserving positivity, and (4) complete positivity, is
\[ \mathcal{E}(\rho) = \sum_i A_i \rho A_i^\dagger, \]  
where the system operators \( A_i \) must satisfy \( \sum_i A_i^\dagger A_i \leq I \). (3)

Then, the entanglement fidelity is defined by
\[ F_\varepsilon(\rho, \mathcal{E}) = \frac{\sum_i |\text{Tr}[A_i \rho]|^2}{\text{Tr}[\mathcal{E}(\rho)]}. \]  

When \( \mathcal{E} \) is trace preserving, or \( \text{Tr}[\mathcal{E}(\rho)] = 1 \) like in the case of quantum teleportation, eqs. (3) and (5) are reduced to
\[ \rho \rightarrow \mathcal{E}(\rho), \]
\[ F_\varepsilon(\rho, \mathcal{E}) = \sum_i |\text{Tr}[A_i \rho]|^2. \]

Hereafter, we restrict ourselves to the case when \( \mathcal{E} \) is trace-preserving. Then, we employ \( E_i \) instead of \( A_i \) in order to emphasize that we consider a special case.

Let us calculate the entanglement fidelity of the phase flip channel, as an example. The channel is described by
\[ \mathcal{E}_{\text{pf}}(\rho) = pp + (1-p)\sigma z \rho \sigma z^\dagger, \]
where \( 1-p \) is the probability that the state of a qubit flips from \( |1\rangle \) to \(-|1\rangle \). The entanglement fidelity
\[ F_\varepsilon(I, \mathcal{E}_{\text{pf}}) = p, \]
where \( I = I/2 \), is obtained. We employ eq. (6) for evaluating the effect of \( T_2 \) relaxation to the entanglement fidelity in §3.7.

We need to perform “quantum process tomography”, a procedure for obtaining \( E_i \), to calculate the entanglement fidelity. The idea of quantum process tomography is as follows. Let us assume that \( \rho \) is a \( d \times d \) matrix, then the map \( \rho \rightarrow \mathcal{E}(\rho) \) should generally be described with \( d^2 \) independent real parameters. When the map is trace-preserving, this gives \( d^2 \) additional constraints. Therefore, a trace-preserving \( \mathcal{E} \) is described with \( d^4 - d^2 \) independent real parameters. When we prepare \( d^2 \) independent input states and measure all output states, the linearity of quantum mechanics guarantees to determine all parameters.

We, now, consider the case of \( d = 2 \). To determine \( E_i \) from measurements, \( E_i \) is parameterized as
\[ E_i = \sum_m e_{im} E_m, \]
with real numbers, \( e_{im} \), where
\[ E_1 = I, \quad E_2 = \sigma_x, \quad E_3 = -i\sigma_y, \quad E_4 = \sigma_z \]
are selected for convenience. Then, \( \mathcal{E} \) is rewritten as
\[ \mathcal{E}(\rho) = \sum_{mn} \chi_{mn} E_m \rho E_n^\dagger, \]
where \( \chi_{mn} = \sum_i e_{im} e_{in}^* \). We can accomplish our goals by determining \( \chi \) from measurements.

Four \((= d^2)\) independent \( \rho_i \)
\[ \rho_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \]
\[ \rho_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \rho_4 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \]
are selected. Then, \( \mathcal{E}(\rho_i) \) is parameterized as
\[ \mathcal{E}(\rho) = \sum_k \lambda_{jk} \rho_k = \begin{pmatrix} \lambda_{j1} & \lambda_{j2} \\ \lambda_{j3} & \lambda_{j4} \end{pmatrix}, \]
where \( \lambda_{jk} \) can be determined from measurements. Note that there are only three independent \( \lambda_{jk} \) for each \( j \), because of the trace preserving condition. We introduce \( \hat{\rho}_{jk}^{mn} \) which is defined by
\[ \hat{\rho}_{jk}^{mn} = \sum_k \chi_{mn} \rho_k. \]
Combining eqs. (8)–(10), we obtain
\[ \sum_k \sum_{mn} \chi_{mn} \hat{\rho}_{jk}^{mn} = \lambda_{jk}. \]

Then, \( \chi \) is obtained by solving eq. (11). In our particular case, eq. (11) is
\[ \begin{pmatrix} \chi_{11} + \chi_{14} + \chi_{41} + \chi_{44} & \chi_{12} + \chi_{13} + \chi_{42} + \chi_{43} & \chi_{21} + \chi_{24} + \chi_{31} + \chi_{34} & \chi_{22} + \chi_{23} + \chi_{32} + \chi_{33} \\ \chi_{12} - \chi_{13} + \chi_{42} - \chi_{43} & \chi_{11} - \chi_{14} + \chi_{41} - \chi_{44} & \chi_{22} - \chi_{23} + \chi_{32} - \chi_{33} & \chi_{21} - \chi_{24} + \chi_{31} - \chi_{34} \\ \chi_{21} + \chi_{24} - \chi_{31} - \chi_{34} & \chi_{22} + \chi_{23} - \chi_{32} - \chi_{33} & \chi_{11} + \chi_{14} - \chi_{41} - \chi_{44} & \chi_{12} + \chi_{13} - \chi_{42} - \chi_{43} \\ \chi_{22} - \chi_{23} - \chi_{32} + \chi_{33} & \chi_{21} - \chi_{24} - \chi_{31} + \chi_{34} & \chi_{12} - \chi_{13} - \chi_{42} + \chi_{43} & \chi_{11} - \chi_{14} - \chi_{41} + \chi_{44} \end{pmatrix} \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{21} & \lambda_{22} \\ \lambda_{13} & \lambda_{14} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44} \end{pmatrix} = \begin{pmatrix} \lambda_{j1} & \lambda_{j2} \\ \lambda_{j3} & \lambda_{j4} \end{pmatrix}. \]

We are also able to calculate \( \chi \) with
\[ \chi = \Lambda \begin{pmatrix} \mathcal{E}(\rho_1) \\ \mathcal{E}(\rho_2) \\ \mathcal{E}(\rho_3) \\ \mathcal{E}(\rho_4) \end{pmatrix} \Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{12} & \lambda_{21} & \lambda_{23} & \lambda_{24} \\ \lambda_{13} & \lambda_{14} & \lambda_{31} & \lambda_{32} \\ \lambda_{14} & \lambda_{13} & \lambda_{41} & \lambda_{42} \end{pmatrix} \begin{pmatrix} \lambda_{j1} & \lambda_{j2} & \lambda_{j3} & \lambda_{j4} \end{pmatrix}. \]
We employ Gaussian pulses, of which pulse widths are hydrogen Larmor frequency is approximately 500 MHz. We employ a JEOL ECA-500 NMR spectrometer, whose out irreversible detection experimentally with an NMR-QC.

3.1 Sample and spectrometer

3. Experiment

2π = 15.8 kHz are measured from the spectrum. Here, ω0q, denotes the Larmor frequency of qubit i. The scalar couplings, J12/2π = 34.8 Hz between qubit 1–2 and J23/2π = 53.8 Hz between qubit 2–3, are also measured, as shown in the upper panel of Fig. 3(b). The scalar coupling between qubit 1–3, J13, is too small to be identified from the spectrum. The scalar couplings, Jd1/2π = 130.0 Hz between qubit 1 and the three protons in the methyl group and Jd2/2π = 145.5 Hz between qubit 2 and the α-proton, are also measured, as shown in the lower panel of Fig. 3(b). We note that rapid exchange of the two amine protons and the single carboxyl proton (see Fig. 3) with the deuterated solvent makes them inactive as qubits (spins).

Fig. 3. Structure of l-alanine and spectra. (a) The whole spectrum is shown when protons are decoupled. (b) The peaks, corresponding qubit 1–3, are enlarged. The upper (lower) panel is with (without) hydrogen decoupling. The strengths (J12, J23, Jd1, Jd2) of the scalar couplings are measured here. See the text for more details.

3.2 Hamiltonian

The Hamiltonian of l-alanine in the individual rotating frame, when protons are decoupled, is approximated by

\[
\mathcal{H} = J_{12}I_{12}^z + J_{23}I_{23}^z,
\]

where

\[
I_{12}^z = I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I,
\]

\[
I_{23}^z = I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I.
\]

The scalar coupling between qubit 1–3 is ignored in eq. (13) since J13 ≪ J12, J23.

The Hamiltonian without decoupling protons is approximated by

\[
\mathcal{H}_d = \mathcal{H} + \mathcal{H}_{d1} + \mathcal{H}_{d2}.
\]
where
\[ \mathcal{H}_{d1} = J_{d1} (I_x \otimes I \otimes I + I \otimes I_x \otimes I + I \otimes I \otimes I_x) \otimes I \]
\[ \mathcal{H}_{d2} = J_{d2} I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \]
\[ \mathcal{H}_{d1} \text{ and } \mathcal{H}_{d2} \text{ in eq. (14) are the scalar couplings between qubit 1 and the three protons in the methyl group and between qubit 2 and the \( \alpha \)-proton, respectively.}

3.3 Detector

We employ the three protons in the methyl group and the \( \alpha \)-proton, shown in Fig. 3(a), as a “detector” that interact with qubit 1 and 2 through the scalar couplings \( \mathcal{H}_{d1} \) and \( \mathcal{H}_{d2} \) in eq. (14). When a heteronuclear decoupling technique is (not) operating, the scalar couplings are (not) nulled. It implies that the “detector” can be switched on and off.

The unitary operator \[ \exp[-i(\mathcal{H}_{d1} + \mathcal{H}_{d2})\tau] \] represents a “detector”, is realized as follows. Here \( \tau \) is a period while the “detector” is on.
\[ \exp[-i(\mathcal{H}_{d1} + \mathcal{H}_{d2})\tau] = U_2(\beta + \pi, \pi) \exp[-i\mathcal{H}(\tau + \tau')]U_2(\beta, \pi) \exp(-i\mathcal{H}_{d2}\tau) \]
where \( U_i(\phi, \theta) \) is an operator \[ \exp[-i(\cos \phi I_x + \sin \phi I_y)] \] acting on qubit \( i \). In NMR quantum computing, the operator \( U_i(\phi, \theta) \) is realized by applying a rf pulse, where \( \phi \) is controlled by its phase and \( \theta \) is done by its strength and pulse width. The operators \[ \exp(-i\mathcal{H}_{d2}\tau) \] can be obtained by free time evolutions of the period \( \tau \) with and without decoupling protons, respectively. The operation \[ U_2(\beta + \pi, \pi) \exp[-i\mathcal{H}(\tau + \tau')]U_2(\beta, \pi) \exp(-i\mathcal{H}_{d2}\tau) \] is for nulling the scalar couplings between qubit 1–2 and qubit 2–3. We ignore the scalar coupling between qubit 1–3 because of its smallness. One of the detector qubit in Fig. 2 is replaced with three protons in the methyl group, and thus the realized unitary operator \[ \exp[-i(\mathcal{H}_{d1} + \mathcal{H}_{d2})\tau] \] is slightly different from eq. (2). However, this difference is not important.

3.4 Pulse sequence

We discuss a NMR pulse sequence implementing the quantum teleportation algorithm with L-alanine molecule.

In the first step, we modify the quantum circuit shown in Fig. 1 since \( J_{13} \) is too small to implement a control-NOT gate between qubit 1 and 3. A standard technique to realize a control-NOT gate between two qubits that have no direct interaction
\[ \text{CNOT}_{13} = \text{SWAP}_{12} \text{CNOT}_{23} \text{SWAP}_{12} \]
is employed for realizing \( \text{CNOT}_{13} \). Here, \( \text{CNOT}_j \) denotes a control-NOT gate (a control qubit = qubit \( i \) and a target qubit = qubit \( j \)) and \( \text{SWAP}_{12} = \text{CNOT}_{12} \text{CNOT}_{32} \text{CNOT}_{12} \) swaps qubit 1 and 2. We omit the swap gate after \( \text{CNOT}_{23} \) since we can leave qubit 1 and 2 swapped. Therefore, the block C in Fig. 1 is modified as follows.

In the second step, we realize an Hadamard gate and a control-NOT gate by employing standard techniques in NMR quantum computing. It is important to compensate an unintended phase shift caused by the Bloch–Siegert effect by adjusting phases of subsequent pulses. We note that this phase correction can easily be as large as 1 rad, or more. We also note that one spectrum is obtained by averaging over four spectra corresponding to four different initial states of qubit 1 and apply the pulse sequence after a Gaussian \( \pi/2 \) pulse applied to qubit 2.

In Fig. 4, we have only one peak at \( \omega_{02} - J_{12}/2 - J_{23}/2 \) in the spectrum of the pseudopure state, while we have four peaks at \( \omega_{02} \pm J_{12}/2 \pm J_{23}/2 \) in the spectrum of the thermal state. The positive peak of the pseudopure state indicates that qubit 2 is \( |0 \rangle \). The frequency of the peak informs the state of the other qubits as follows. The peak at \( \omega_{02} - J_{12}/2 - J_{23}/2 \), \( \omega_{02} - J_{12}/2 + J_{23}/2 \), \( \omega_{02} + J_{12}/2 - J_{23}/2 \), \( \omega_{02} + J_{12}/2 + J_{23}/2 \), corresponds to the state \( |\text{qubit 1, qubit 3}\rangle = |0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle \), respectively. Therefore, the spectrum in Fig. 4 indicates that the state of \( |000\rangle \) is obtained.

3.5 Initialization

Qubit 1–3 are set initially in the pseudopure state \( |000\rangle \) by employing a spatial averaging method with field gradient pulses. All considerations discussed in §3.4 are also taken into account for making the pulse sequence which creates \( |000\rangle \) state. The pulse sequence consists of 26 pulses, 4 pulsed field gradients, and 12 free evolution periods and takes about \( t_{ps} = 95 \text{ ms} \).

We evaluate a created pseudopure state by applying a Gaussian \( \pi/2 \) pulse on qubit 2 \( |U_2(\pi/2, \pi/2)\rangle \), as shown in Fig. 4. We have only one peak at \( \omega_{02} - J_{12}/2 - J_{23}/2 \) in the spectrum of the pseudopure state, while we have four peaks at \( \omega_{02} \pm J_{12}/2 \pm J_{23}/2 \) in that of the thermal state. The positive peak of the pseudopure state indicates that qubit 2 is \( |0 \rangle \). The frequency of the peak informs the state of the other qubits as follows. The peak at \( \omega_{02} - J_{12}/2 - J_{23}/2 \), \( \omega_{02} - J_{12}/2 + J_{23}/2 \), \( \omega_{02} + J_{12}/2 - J_{23}/2 \), \( \omega_{02} + J_{12}/2 + J_{23}/2 \), corresponds to the state \( |\text{qubit 1, qubit 3}\rangle = |0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle \), respectively. Therefore, the spectrum in Fig. 4 indicates that the state of \( |000\rangle \) is obtained.

3.6 Quantum teleportation

We perform a quantum teleportation experiment with the pulse sequence discussed in §3.4. We take the following four initial states of qubit 1 and apply the pulse sequence to the system.
I've got to jump.
perfect. Note that not only imperfections in the pulse sequence but also $T_1$ relaxation of qubit 3 causes decrease of the entanglement fidelity. The time requires for quantum teleportation operation is $t_{ps} = 95$ ms and is not negligible compared with $T_1 (3) = 0.41$ s of qubit 3, and thus $p$ in eq. (6) is $0.5 + \exp[-t_{ps}/T_1 (3)]/2 \approx 0.9$. We observe that the entanglement fidelities are almost constant despite of disturbances by the “detector”'s. $\text{Tr}(\mathcal{E})$'s are close to unity as expected, although there are some errors.

4. Summary

We introduce a well defined detector (or, more precisely a quantum mechanical interface to a detector) to quantum teleportation operation unlike other realizations. Then, we realize “quantum teleportation without irreversible detection” discussed by Braunstein.\(^4\) Our experiments help us to understand the mechanism of quantum teleportation with a very concrete example. We also show technical details of experiment.

Acknowledgments

We would like to thank Mikio Nakahara and Shogo Tanimura for discussions, Toshie Minematsu for assistance in NMR operations, and Katsuo Asakura for assistance in NMR pulse programming.

---

<table>
<thead>
<tr>
<th>$\tau$ (ms)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>4.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_e(I_e, E)$</td>
<td>0.78</td>
<td>0.78</td>
<td>0.79</td>
<td>0.77</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>$\text{Tr}(\mathcal{E})$</td>
<td>1.06</td>
<td>1.09</td>
<td>1.04</td>
<td>1.10</td>
<td>1.10</td>
<td>1.02</td>
</tr>
</tbody>
</table>

---

11) R. B. Griffiths and C.-S. Niu: Phys. Rev. Lett. 76 (1996) 3228; Note that a control-U gate, if its control qubit is to be measured in the standard basis, leads the same final outcome regardless whether its control qubit is measured either before or after the gate is executed.